Online Appendix of Supplementary Material
for
Can Stricter Bankruptcy Laws Discipline Capital Investment? Evidence from the U.S. Airline Industry

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September 30, 2022

Contents:

A.1. CCP Representation

To apply the two-step procedure requires us to find a way to represent the analytic CCPs as explicit functions of the structural parameters and the empirical CCPs. Consider again the ex ante value function for firm i in state k:

$$V_{i,k} = \frac{1}{\rho + N\lambda_a + N\lambda_b + \gamma}\left(u_{i,k} + \gamma V_{i,l(demand,k)} + \lambda_a \mathbb{E}\left[V_{i,l(i,k;a)}\right] + \lambda_b \mathbb{E}\left[V_{i,l(i,k;b)}\right] + \sum_{i' \neq i} \lambda_a \mathbb{E}\left[V_{i,l(i',k;a)}\right] + \sum_{i' \neq i} \lambda_b \mathbb{E}\left[V_{i,l(i',k;b)}\right]\right)$$

and apply what we know of the expectation terms

$$\begin{aligned} \mathbb{E}\left[V_{i,l(i,k;r)}\right] &= \mathbb{E} \max_{j \in X_{i,k;r}} \left\{V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij}\right\} \\ \mathbb{E}\left[V_{i,l(i',k;r)}\right] &= \sum_{j \in X_{i',k;r}} \sigma_{i',j,k} V_{i,l(i',j,k)} \end{aligned}$$

to arrive at

$$V_{i,k} = \frac{1}{\rho + N\lambda_a + N\lambda_b + \gamma}\left(u_{i,k} + \gamma V_{i,l(demand,k)} + \sum_{r \in \{a,b\}} \lambda_r \left(\mathbb{E} \max_{j \in X_{i,k;r}} \left\{V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij}\right\} + \sum_{i' \neq i} \sum_{j \in X_{i',k;r}} \sigma_{i',j,k} V_{i,l(i',j,k)}\right)\right) \tag{1}$$

i. Incumbents

Applying Proposition 2 of Arcidiacono et al. (2016), we can write the Emax term as follows:

$$\mathbb{E} \max_{j \in X_{i,k;r}} \left\{V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij}\right\} = \gamma_{eul} + \psi_{j',k} + V_{i,l(i,j',k)} - \log(\sigma_{j',k})$$

for any choice $j' \in X_{i,k;r}$. For incumbents facing a capacity adjustment opportunity, we can choose $j' = exit$ and normalize the continuation value of $exit$ to 0,[1] giving us

$$\mathbb{E} \max_{j \in X_{i,k;a}} \left\{V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij}\right\} = \gamma_{eul} + \psi_{exit,k} - \log(\sigma_{exit,k}) \tag{2}$$

The same proposition allows us to compare the value functions of different states, as reflected in Proposition 1 of Arcidiacono et al. (2016) below:

$$\begin{aligned} \gamma_{eul} + \psi_{j,k} + V_{i,l(i,j,k)} - \log(\sigma_{j,k}) &= \gamma_{eul} + \psi_{j',k} + V_{i,l(i,j',k)} - \log(\sigma_{j',k}) \\ V_{i,l(i,j,k)} &= V_{i,l(i,j',k)} + \psi_{j',k} - \psi_{j,k} + \log(\sigma_{j,k}) - \log(\sigma_{j',k}) \end{aligned}$$

where j and $j'$are elements of the same choice set $X_{i,k;r}$. To compare value functions across choice sets, suppose that player i in state k will always have a continuation choice, j^*, that

[1] Note that the adjustment cost of exit is state-specific, so while leaving the industry is worth zero going forward, the context in which a carrier exits (e.g. liquidation, merger, etc.) is allowed to matter.

does not change the state. In other words, players can always choose to do nothing, regardless of the type of move opportunity that arrives. Combining the continuation choice with the second expression above gives us the following two equalities:

$$\begin{aligned} V_{i,l(i,j_a^*,k)} &= V_{i,l(i,j_a,k)} + \psi_{j_a,k} - \psi_{j_a^*,k} + \log(\sigma_{j_a^*,k}) - \log(\sigma_{j_a,k}) \\ V_{i,l(i,j_b^*,k)} &= V_{i,l(i,j_b,k)} + \psi_{j_b,k} - \psi_{j_b^*,k} + \log(\sigma_{j_b^*,k}) - \log(\sigma_{j_b,k}) \end{aligned}$$

where the j choices have subscripts to indicate their relevant choice sets. Recognizing that $V_{i,l(i,j_a^*,k)} = V_{i,l(i,j_b^*,k)}$, we can write

$$V_{i,l(i,j_a,k)} + \psi_{j_a,k} - \psi_{j_a^*,k} + \log(\sigma_{j_a^*,k}) - \log(\sigma_{j_a,k}) = V_{i,l(i,j_b,k)} + \psi_{j_b,k} - \psi_{j_b^*,k} + \log(\sigma_{j_b^*,k}) - \log(\sigma_{j_b,k})$$

which, assuming there is no instantaneous cost to choosing the status quo, simplifies to

$$V_{i,l(i,j_b,k)} = V_{i,l(i,j_a,k)} + \psi_{j_a,k} + \log(\sigma_{j_a^*,k}) - \log(\sigma_{j_a,k}) - \psi_{j_b,k} - \log(\sigma_{j_b^*,k}) + \log(\sigma_{j_b,k})$$

If we set $j_a = exit$ and apply the the normalization of equation (20), we get

$$V_{i,l(i,j_b,k)} = \psi_{exit,k} + \log(\sigma_{j_a^*,k}) - \log(\sigma_{exit,k}) - \psi_{j_b,k} - \log(\sigma_{j_b^*,k}) + \log(\sigma_{j_b,k})$$

For an incumbent facing a bankruptcy adjustment opportunity, we can substitute this expression into Proposition 1 and cancel terms to arrive at

$$\mathbb{E} \max_{j \in X_{i,k;b}} \left\{ V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij} \right\} = \gamma_{eul} + \psi_{exit,k} + \log(\sigma_{j_a^*,k}) - \log(\sigma_{exit,k}) - \log(\sigma_{j_b^*,k}) \quad (3)$$

Next consider the value to player i of an opponent's choice, which moves the state from k to $k' \equiv l(i', j, k)$. Note that the value to player i of being in state k' does not depend on how player i arrived in that state. Therefore, if we again let j^* represent a continuation choice for player i, such that $l(i, j^*, k') = k'$, then we have $V_{i,l(i',j,k)} = V_{i,k'} = V_{i,l(i,j^*,k')}$. Proposition 3 of Arcidiacono et al. (2016) applies this equivalence, allowing us to re-write Proposition 1 as follows:

$$V_{i,l(i',j,k)} = V_{i,l(i,j^*,k')} = V_{i,l(i,j',k')} + \psi_{j',k'} - \psi_{j^*,k'} + \log(\sigma_{j^*,k'}) - \log(\sigma_{j',k'})$$

where j' is any of player i's choices in state k', and we can again set $\psi_{j^*,k'} = 0$. In addition, whenever player i's choice set in state k' includes both *exit* and a continuation choice j^* (i.e. for a capacity adjustment decision), we can substitute $V_{i,l(i,exit,k')} = 0$ for $V_{i,l(i,j',k')}$ to get an even tidier result:

$$V_{i,l(i',j,k)} = \psi_{exit,l(i',j,k)} + \log(\sigma_{j^*,l(i',j,k)}) - \log(\sigma_{exit,l(i',j,k)}) \tag{4}$$

where I have applied $k' \equiv l(i', j, k)$ to make it clear that we still have an opponent's move in view. The same expression applies to moves by nature. Expressions (12)-(14) will be valid for all states in which player i is an incumbent. Substituting them into (11) expresses each value function in terms of CCPs and parameters.

$$\begin{aligned} V_{i,k}(\rho + N\lambda_a + N\lambda_b + \gamma) \;=\;& u_{i,k} + \gamma\left[\psi_{exit,l(demand,k)} - \log(\sigma_{exit,l(demand,k)}) + \log(\sigma_{j_a^*,l(demand,k)})\right] \\ +\;& \lambda_a\left[\gamma_{eul} + \psi_{exit,k} - \log(\sigma_{exit,k})\right] \\ +\;& \lambda_b\left[\gamma_{eul} + \psi_{exit,k} - \log(\sigma_{exit,k}) + \log(\sigma_{j_a^*,k}) - \log(\sigma_{j_b^*,k})\right] \\ +\;& \sum_{r\in\{a,b\}} \lambda_r \left(\sum_{i'\neq i} \sum_{j\in X_{i',k;r}} \sigma_{i',j,k}\left[\psi_{exit,l(i',j,k)} - \log(\sigma_{exit,l(i',j,k)}) + \log(\sigma_{j_a^*,l(i',j,k)})\right] \right) \end{aligned}$$

ii. Potential Entrants

For potential entrants, *exit* is not an option, so we must apply another substitution in order to eliminate value functions on the right-hand side. As before, apply Proposition 2 to get

$$\mathbb{E} \max_{j\in X_{i,k;a}} \left\{V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij}\right\} = \gamma_{eul} + \psi_{j',k} + V_{i,l(i,j',k)} - \log(\sigma_{j',k})$$

but now let $j' = 1$, the choice to enter with the lowest possible capacity, and define $k^* \equiv l(i, 1, k)$. Then, since player i is an incumbent in state k^*, simply apply equation (14) to $V_{i,l(i,j',k)}$ to get an expression for the future value of player i's move:

$$\mathbb{E} \max_{j\in X_{i,k;a}} \left\{V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij}\right\} = \gamma_{eul} + \psi_{1,k} + \left[\psi_{exit,k^*} + \log(\sigma_{j_a^*,k^*}) - \log(\sigma_{exit,k^*})\right] - \log(\sigma_{1,k}) \tag{5}$$

To represent the future values of opponents' moves, apply Proposition 1 as before:

$$V_{i,l(i',j,k)} = V_{i,l(i,j_a^*,k')} \;=\; V_{i,l(i,j',k')} + \psi_{j',k'} - \psi_{j_a^*,k'} + \log(\sigma_{j_a^*,k'}) - \log(\sigma_{j',k'})$$

Choose $j' = 1$ and apply equation (14) again to get

$$V_{i,l(i',j,k)} = \left[\psi_{exit,l(i,1,k')} + \log(\sigma_{j_a^*,l(i,1,k')}) - \log(\sigma_{exit,l(i,1,k')})\right] + \psi_{1,k'} + \log(\sigma_{j_a^*,k'}) - \log(\sigma_{1,k'}) \tag{6}$$

Substitute (15) and (16) into (11) to get expressions for each value function

$$
\begin{gathered}
V_{i,k}\left(\rho + N\lambda_a + (N-1)\lambda_b + \gamma\right) = \\
u_{i,k} + \gamma\left[\left[\psi_{exit,l(i,1,k'')} + \log(\sigma_{j_a^*,l(i,1,k'')}) - \log(\sigma_{exit,l(i,1,k'')})\right] + \psi_{1,k''} + \log(\sigma_{j_a^*,k''}) - \log(\sigma_{1,k''})\right] \\
+\lambda_a\left[\gamma_{eul} + \psi_{1,k} + \left[\psi_{exit,k^*} + \log(\sigma_{j_a^*,k^*}) - \log(\sigma_{exit,k^*})\right] - \log(\sigma_{1,k})\right] \\
+\textstyle\sum_{r\in\{a,b\}}\lambda_r\left(\sum_{i'\neq i}\sum_{j\in X_{i',k;r}}\sigma_{i',j,k}\left[\left[\psi_{exit,l(i,1,k')} + \log(\sigma_{j_a^*,l(i,1,k')}) - \log(\sigma_{exit,l(i,1,k')})\right] + \psi_{1,k'} + \log(\sigma_{j_a^*,k'}) - \log(\sigma_{1,k'})\right]\right)
\end{gathered}
$$

where $k'' \equiv l(demand, k)$, $k' \equiv l(i', j, k)$, and I have prohibited potential entrants from filing for bankruptcy.

A.2. Overview of Distributions

The following is meant as a refresher on the PDFs and CDFs of Poisson and exponential distributions. Poisson is a discrete distribution with PMF

$$P(x) = \frac{\exp(-\mu)\mu^x}{x!}$$

where $x = 0, 1, 2, ...$ and the mean and variance are both μ. Exponential is a continuous distribution with PDF

$$f(t) = \lambda \exp(-\lambda t)$$

where $t \geq 0$ and the mean and variance are both $\frac{1}{\lambda}$. The exponential CDF is

$$F(t) = 1 - \exp(-\lambda t)$$

If the Poisson describes the number of occurrences per unit of time, then the exponential describes the duration between occurrences. That is, the Poisson rate describes how many events should occur, on average, per unit of time. If λt events occur in t units of time, then the probability that no events occur in an interval of t is

$$P(0; \mu = \lambda t) = \frac{\exp(-\lambda t)\, (\lambda t)^0}{0!} = \exp(-\lambda t)$$

Therefore, the probability that an event has not occurred after t time has passed is $1 - \exp(-\lambda t) = F(t)$.

Finally, let's match the exponential distribution to the intensity matrix of a Markov jump process. Following Chapter 3 of Hoel et al. (1986), a jump process is a sequence $X(t)$ that describes the state of a system at time t in the following way:

$$X(t) = \begin{cases} x_0, & 0 \leq t < \tau_1 \\ x_1, & \tau_1 \leq t < \tau_2 \\ x_2, & \tau_2 \leq t < \tau_3 \\ \vdots \end{cases}$$

A pure jump process is one that is non-explosive, that is, one for which $\lim_{n\to\infty} \tau_n = \infty$. The jump times and associated states are random. If the process reaches an absorbing state, it remains there forever, whereas if the process reaches a non-absorbing state k, it remains there for some length of time t, which is distributed according to $F_k(t)$. After t elapses, the process jumps from state k to state l with probability Q_{kl} (define $Q_{kk} = 0$). Moreover, the time and state events are independent. Hence, if we consider a pure jump process beginning in state k at time 0, then

$$\mathbb{P}\left[\tau_1 \leq t,\ X(\tau_1) = l \mid x_0 = k\right] = F_k(t) Q_{kl}$$

Referencing Chapter 5 of Hoel et al. (1971), Hoel et al. (1986) point out that a pure jump process is Markovian if and only if $F_x(t)$ is exponential for every non-absorbing state x. Let $\mathbb{P}_x$ denote the probability of an event conditional on the current state being x. Then the Markov property means

$$\mathbb{P}_x\left[\tau_1 > t + s,\ \mid \tau_1 > s\right] = \mathbb{P}_x\left[\tau_1 > t\right]$$

Further, let $\mathbb{P}_{xy}(t)$ denote the probability that a process beginning in state x is in state y at time t, and let $\mathbb{P}_{xy}(0) = \delta_{xy}$, where

$$\delta_{xy} = \begin{cases} 1, & y = x \\ 0, & y \neq x \end{cases}$$

Then the Markov property also implies

$$\mathbb{P}_{xy}(t+s) = \sum_z \mathbb{P}_{xz}(t)\mathbb{P}_{zy}(s)$$

To relate all of this to the intensity matrix I use in the paper, let q_x be the parameter that defines the exponential distribution F_x. Since I always have to be reminded, that means

$$F_x(t) = 1 - \exp(-q_x t)$$

and

$$\mathbb{P}_x\left[\tau_1 \geq t\right] = 1 - F_x(t) = \exp(-q_x t)$$

We can describe $\mathbb{P}_{xy}(t)$ as the integral over all possible jumps from time from 0 to t that take the process from state x to state z, each weighted by the probability of thereafter transitioning from state z to state y. This yields the Chapman-Kolmogorov equation.

$$\begin{aligned} \mathbb{P}_{xy}(t) &= \delta_{xy}\left(1 - F_x(t)\right) + \int_0^t f_x(s)\left(\sum_{z \neq x} Q_{xz}\mathbb{P}_{zy}(t-s)\right) \mathrm{d}s \\ &= \delta_{xy}\exp(-q_x t) + \int_0^t q_x \exp(-q_x s)\left(\sum_{z \neq x} Q_{xz}\mathbb{P}_{zy}(t-s)\right) \mathrm{d}s \end{aligned}$$

To get the infinitesimal parameters of the intensity matrix, first replace s with $t-s$ to get [2]

$$\begin{aligned}\mathbb{P}_{xy}(t) &= \delta_{xy}\exp(-q_x t) + \int_0^t q_x \exp(-q_x t + q_x s)\left(\sum_{z\neq x} Q_{xz}\mathbb{P}_{zy}(s)\right)\mathrm{d}s \\ &= \delta_{xy}\exp(-q_x t) + q_x\exp(-q_x t)\int_0^t \exp(q_x s)\left(\sum_{z\neq x} Q_{xz}\mathbb{P}_{zy}(s)\right)\mathrm{d}s\end{aligned}$$

and then differentiate with respect to t to get

$$\begin{aligned}\mathbb{P}'_{xy}(t) &= -q_x\delta_{xy}\exp(-q_x t) - q_x^2\exp(-q_x t)\int_0^t \exp(q_x s)\left(\sum_{z\neq x} Q_{xz}\mathbb{P}_{zy}(s)\right)\mathrm{d}s + q_x\exp(-q_x t)\left[\exp(q_x t)\left(\sum_{z\neq x} Q_{xz}\mathbb{P}_{zy}(t)\right)\right] \\ &= -q_x\mathbb{P}_{xy}(t) + q_x\left(\sum_{z\neq x} Q_{xz}\mathbb{P}_{zy}(t)\right)\end{aligned}$$

which at $t=0$ reduces to

$$\mathbb{P}'_{xy}(0) = -q_x\delta_{xy} + q_x Q_{xy}$$

Define $q_{xy} \equiv \mathbb{P}'_{xy}(0)$, and we can write the elements of the intensity matrix in a familiar way

$$q_{xy} = \begin{cases} -q_x, & y = x \\ q_x Q_{xy}, & y \neq x \end{cases}$$

and since $\sum_y Q_{xy} = 1$, we know that $\sum_y q_{xy} = q_x = -q_{xx}$. Recall that each non-diagonal element of the intensity matrix is the instantaneous probability (hazard rate) of transitioning from one state to another:

$$q_{kl} = \lim_{h\to 0}\frac{\mathbb{P}\left[X_{t+h} = l | X_t = k\right]}{h}$$

and the sum of these probabilities represents the rate at which the process leaves state k. Therefore, the CDF of the duration spent in state x is given by

$$F_x(t) = 1 - \exp(-t\sum_y q_{xy})$$

[2]Use Liebniz rule:

$$\frac{\partial}{\partial t}\left(\int_{a(t)}^{b(t)} f(t,s)\mathrm{d}s\right) = f(t,b(t))b'(t) - f(t,a(t))a'(t) + \int_{a(t)}^{b(t)} f_t(t,s)\mathrm{d}s$$

In this case, $a(t) = 0$ and $b(t) = t$

References

[1] Arcidiacono, P., P. Bayer, J. Blevins, and P. Ellickson (2016). "Estimation of Dynamic Discrete Choice Models in Continuous Time with an Application to Retail Competition," Review of Economic Studies 83(3): 889-931.

[2] Hoel, P. G., S. C. Port, and C. J. Stone (1971). *Introduction to Probability Theory*, Volume 12. Houghton Mifflin Boston.

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